NASA
Technical Memorandum 107241

Army Research Laboratory
Technical Report ARL–TR–1117

A Computer Program for the Computation of Running Gear Temperatures Using Green's Function

S. Koshigoe, J.W. Murdock, and L.S. Akin
Gearesearch Associates
Banning, California

D.P. Townsend
Lewis Research Center
Cleveland, Ohio

Prepared for the
Seventh International Power Transmission and Gearing Conference
sponsored by the American Society of Mechanical Engineers
San Diego, California, October 6–9, 1996

National Aeronautics and
Space Administration

U.S. ARMY
RESEARCH LABORATORY

A COMPUTER PROGRAM FOR THE COMPUTATION OF RUNNING GEAR TEMPERATURES USING GREEN'S FUNCTION

S. Koshigoe, J. W. Murdock,* and L. S. Akin*****
Gearesearch Associates
750 Indian Wells Rd.
Banning , California 92220-5308

D. P. Townsend***
National Aeronautics and Space Administration
Lewis Research Center
Cleveland Ohio 44135

ABSTRACT

A new technique has been developed to study two dimensional heat transfer problems in gears. This technique consists of transforming the heat equation into a line integral equation with the use of Green's theorem. The equation is then expressed in terms of eigenfunctions that satisfy the Helmholtz equation, and their corresponding eigenvalues for an arbitrarily shaped region of interest. The eigenfunction are obtained by solving an intergral equation. Once the eigenfunctions are found, the temperature is expanded in terms of the eigenfunctions with unknown time dependent coefficients that can be solved by using Runge-Kutta methods. The time integration is extremely efficient. Therefore, any changes in the time dependent coefficients or source terms in the boundary conditions do not impose a great computational burden on the user. The method is demonstrated by applying it to a sample gear tooth. Temperature histories at representative surface locatons are given.

INTRODUCTION

The cooling of gears is an important problem that has been studied for a number of years. An early model of oil cooling is given in DeWinter and Block (1972). El-Bayoumy et al. (1989) expands on the model of DeWinter and Block (1972) mainly by noting the importance of the Coriolis force on oil cooling of gears. El-Bayoumy et al. (1989) also develops a finite element model of the gear tooth. We now believe that the finite element method has speed and accuracy limitations and have abandoned this approach in favor of the Green's function method described herein.

In the next section, a new dynamic, accurate and efficient solution method for two dimensional heat transfer problems in gears is described. The solution method consists of transforming the heat equation into a line integral equation with the use of Green's functions with unknown time dependent coefficients. The transformed integral equation is used to obtain the dynamic equations for the time dependent coefficients for each eigenfunction. In order to obtain the eigenvalues and corresponding eigenfunctions, the Helmholtz equation for the eigenfunctions is transformed into a line integral equation by the use of the two-dimensional free space Green's function. The integral equation is discretized into a set of homogenous simultaneous equations. The discretized version of the eigenfunction can be obtained by solving a set of homogenous simultaneous equations. The obtained dynamic equations can be integrated extremely efficiently. Therefore, any changes in the boundary conditions do not impose a great computational burden. In the third section, the computational results and a discussion is presented.

FORMULATION

In this section, an accurate and efficient solution method for solving a time dependent two dimensional heat problems in gears is developed. The temperature field is expanded in terms of the eigenfunctions with unknown time dependent dynamic coefficients. The dynamic equation for the time dependent coefficients of each eigenfunction is obtained by the use of an integral equation.

Consider a gear tooth geometry shown in Fig. 1. The transient heating of the gear tooth is described by

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T \tag{1}$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ and α is the thermal diffusivity. The boundary condition for the left side and top of the gear is given by

$$K\frac{\partial T}{\partial n} = h(\xi)\left(T(\xi) - T_c\right) \tag{2}$$

*Member ASME. **Member AIAA. ***Fellow ASME.

where the equation represents the heat output to cooling oil, $\bar{\xi}$ is the vector coordinate on the boundary, K is the thermal conductivity, n is the outward pointing normal unit vector, h is the heat transfer coefficient and T_c is the temperature of the cooling oil. The boundary condition for loaded side (meshing surface) of the gear (right side in Fig. 1) determines the heat conduction into gear body and is given by

$$K\frac{\partial T}{\partial n} = F_0(\bar{\xi}) \tag{3}$$

where F_0 is the heat flux into the gear and the remainder of the boundary is described by the vanishing normal derivative of the temperature, $\partial T/\partial n = 0$.

Dynamical Equations for Transient Heat Flow

Rather than solve the physical equations directly, as in a finite element method or finite difference method, we develop here a Green's function method that reduces the two-dimensional problem to a one-dimensional line integral over the gear tooth boundary. The line integral equation yields the dynamical equations. This procedure yields a computational advantage over the previous approaches where there is no reduction in the problem dimensionally and the time and spatial integrations are performed simultaneously.

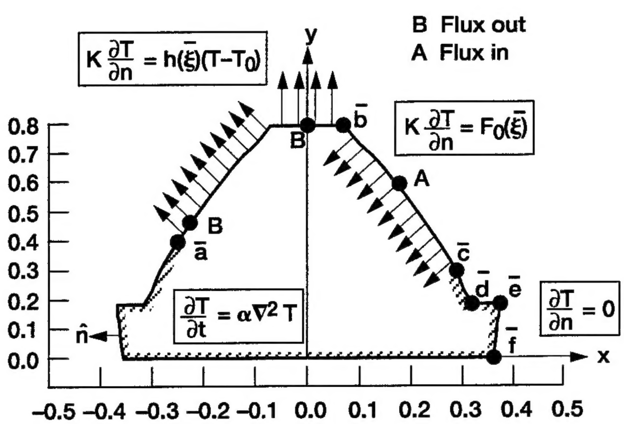

Figure 1.—Simulated gear tooth geometry and all the boundary conditions.

We assume, the following eigenfunctions, ψ_n, and eigenvalues, k_n are known and the eigenfunctions satisfy the Helmholtz equation,

$$\left(\nabla^2 + k_n^2\right)\psi_n = 0 \tag{4}$$

inside the gear tooth with vanishing normal derivative, $\partial\psi_n/\partial n = 0$. (The method for obtaining the eigenvalues and eigenfunctions is given in the following section.) It can be shown that these eigenfunctions are orthogonal. The eigenfunctions will be normalized to unity by $\iint \psi_n^2 dxdy = 1$.

The heat equation is transformed into a line integral equation by using Green's theorem, (Wyld, 1972)

$$\iint\left(T\nabla^2\psi_n - \psi_n\nabla^2 T\right)ds = \int\left(T\frac{\partial\psi_n}{\partial n} - \psi_n\frac{\partial T}{\partial n}\right)dl \tag{5}$$

where ds is the surface element and dl is the line element. Substituting Eqs. (1) and (4) into the left hand side of Eq. (5) yields

$$k_n^2\iint T\psi_n ds + \frac{1}{\alpha}\iint \psi_n\frac{\partial T}{\partial n}ds = \int_{\bar{a}}^{\bar{c}}\psi_n\frac{\partial T}{\partial n}dl \tag{6}$$

where the vectors $\bar{a}$ and $\bar{c}$ in the integration limits are shown in Fig. 1 (a driven gear). Note that a considerable simplification has been accomplished for the line integral on the right hand side of Eq. (5). The only contribution comes from the boundary conditions described by the non-vanishing normal derivatives, Eqs. (2) and (3). Substitution of Eqs. (2) and (3) and expansion of the temperature field in terms of the eigenfunctions as

$$T(\bar{x},t) = \sum_n \eta_n(t)\psi_n(\bar{x}) \tag{7}$$

yield a dynamic equations for the time dependent coefficients, η_n,

$$\dot{\eta}_n + \alpha k_n^2\eta_n = \frac{\alpha}{K}\left[\sum_m A_{mm}\eta_m + B_n + C_n\right] \tag{8}$$

where

$$A_{mm} = \int_{\bar{a}}^{\bar{b}} h(\bar{\xi})\psi_m(\bar{\xi})\psi_n(\bar{\xi})dl \tag{9a}$$

$$B_n = -T_c\int_{\bar{a}}^{\bar{b}} h(\bar{\xi})\psi_n(\bar{\xi})dl \tag{9b}$$

$$C_n = \int_{\bar{b}}^{\bar{c}} F_0(\bar{\xi})\psi_n(\bar{\xi})dl \tag{9c}$$

The line integrals are easily calculated as the boundary conditions are changed. Therefore, the calculation burden for changing the boundary conditions is minimal. These coupled first order equations in Eq. (8) are integrated efficiently with Runge-Kutta method.

Eigenvalues and eigenfunctions

The method for obtaining the eigenvalues and eigenfunctions used to effect the simplification of the previous section is given below. These functions are two dimensional analogs of the trigonometric functions used in Fourier methods.

The eigenvalues, k_n and the corresponding eigenfunctions, ψ_n are obtained through the use of two dimensional free space Green's function (Koshigoe and Tubis 1989),

$$G(\bar{x}|\bar{x}') = \frac{1}{4} H_o^{(1)}(k_n|\bar{x} - \bar{x}'|) \tag{10}$$

where $H_0^{(1)}$ is the Hankel function of the first kind and the Green's function satisfies the inhomogeneous Helmholtz equation,

$$(\nabla^2 + k_n^2) G(\bar{x}|\bar{x}') = -\delta(\bar{x} - \bar{x}') \tag{11}$$

where $\delta(\bar{x})$ is the delta function.

Again utilizing Green's theorem, one obtains

$$\iint \left(G(\nabla'^2 + k_n^2)\psi_n - \psi_n(\nabla'^2 + k_n^2) G \right) ds'$$

$$= \int \left(G(\bar{x}|\bar{\xi}') \frac{\partial \psi_n}{\partial n'} - \psi_n \frac{\partial G(\bar{x}|\bar{\xi}')}{\partial n'} \right) dl' \tag{12}$$

With the use of Eqs. (4) and (11) and vanishing normal derivative boundary condition for the eigenfunctions the integral equation is simplified

$$\psi_n(\bar{x}) + \int \psi_n(\bar{\xi}') \frac{\partial G(\bar{x}|\bar{\xi}')}{\partial n'} dl' = 0 \tag{13}$$

This integral equation yields the value of ψ_n at any location inside the gear tooth when the values of ψ_n on the gear tooth boundary are known. Now let x approach a point, $\bar{\xi}$ on the gear tooth boundary, then Eq. (13) becomes

$$\beta_\xi \psi_n(\bar{\xi}) + P\int \psi_n(\bar{\xi}') \frac{\partial G(\bar{\xi}|\bar{\xi}')}{\partial n'} dl' = 0 \tag{14}$$

where β_ξ is the contribution from the singularity in the integrand (Burton and Miller 1971) and is given by:

$$\beta_\xi = \text{inside angle at the point } \bar{\xi}/2\pi \tag{15}$$

and P designates the Cauchy principal value integral. The eigenfunction, ψ_n is discretized in Eq. (14) and yields a set of simultaneous equations. The eigenvalues, k_n are determined by setting the determinant of the simultaneous equations to zero. Once the eigenvalues are determined, the corresponding eigenfunctions can be obtained through the simultaneous equations. The formulation developed in this section can now be applied to the gear tooth geometry (shown in Fig. 1) and the calculation result discussed.

SAMPLE CALCULATION

The technique developed in the previous section is applied to the gear tooth geometry shown in Fig. 1. Various coordinates are labeled in Fig. 1 in order to specify the key features of the gear tooth geometry. The cooardinates of these points are given in table 1. The physical constants used for the calculation are: the gear thermal diffusivity, $\alpha = 0.452$ ft^2/hr; the thermal conductivity, K = 25 Btu/hr/ft/F; the oil temperature, T_c = 200 F; the heat transfer coefficient, h = 0.34 Btu/sec/ft^2/F follows from DeWinter and Block (1972) and El-Baypoumy et al. (1989).

The heat flux, F_0, along the boundary from the location $\bar{\xi} = \bar{b}$ to $\bar{c}$ (shown in Fig. 1), is given as a function of the distance measured from the point $\bar{b}$ (shown in Fig. 2). This is the heat generated in mesh for a 1-inch wide gear, with a pitch radius of 6 inches, rotating a 10,000 rpm and transmitting 500 hp. We are interested in gear steady-state temperatures that take hundreds of seconds to reach. Hence, the detailed temperature changes that occur as the gear goes in and out of mesh cannot be resolved. Thus, both the heating and the cooling have been averaged over a complete revolution cycle.

Table 1.—Coordinate of Points in Figure 1

Figure 1 location	x, inches	y, inches
$\bar{a}$	–0.197	0.541
$\bar{b}$	.073	.795
$\bar{c}$	.274	.337
$\bar{d}$	.318	.184
$\bar{e}$	.371	.184
$\bar{f}$	.362	.000

The temperature calculation was performed using 21 eigenmodes. (This was found to provide adequate convergence.) The results presented herein were generated on a 486–PC with a total running time, including the costly eigenfunction generation, of less than one hour.

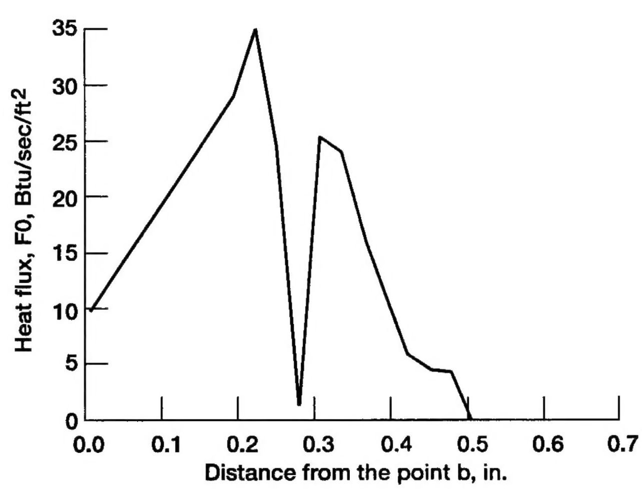

Figure 2.—Time-averaged heat input to gear.

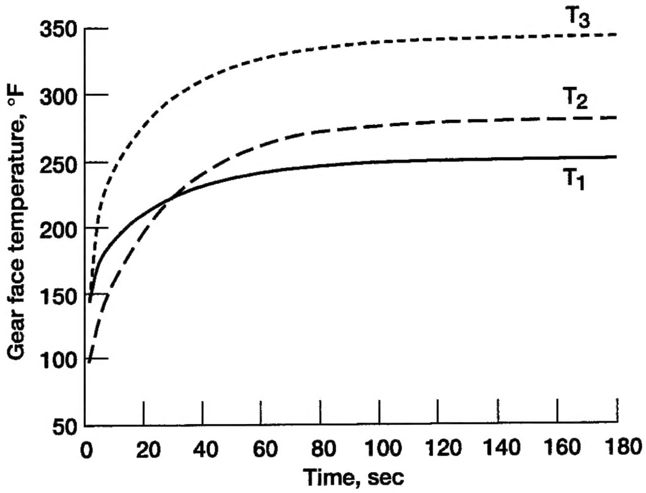

Figure 3.—Gear surface temperature at selected locations.

The eigenvalues and the corresponding eigenfunctions are obtained through the use of discretized version of Eq. (14). The discretization is carried out with 40 line elements and on each element, the second order approximation is used to represent the geometry as well as the eigenfunctions. The temperature calculations at three different locations on the gear tooth boundary, (0.0,0.397) represented by the solid line, (–0.228,0.066) by the dashed line, and (0.174,0.194) by the dotted line are shown in Fig. 3 as functions of time. The curve labeled T1 is centered on the top of the gear at location (0.0, 0.795). T2 on the cooled side of the gear in the region cooled by the oil at (–0.228, 0.464). T3, the highest curve, is on the side heated during mesh, slightly above the pitch point at (0.174, 0.592). Because both the heating and cooling functions have been averaged over a complete gear revolution, these temperatures can best be interpreted as out-of-mesh temperatures.

At the present time the calculation method is hard wired for a single, but representative, problem. Furthermore, we have not yet included the triangular portion of the gear extending to the axis of rotation. Generalization of the method is planned now that its application to a specific problem has been demonstrated. Appendix A "Thermal Analysis of Spur Gears" provides the geometric formulation input.

CONCLUSION

A new technique has been developed to study two-dimensional heating of gears. The computational advantage of this technique over previous approaches using the finite element method or the finite difference method results from two features: First, the problem is reduced from two dimensions to one. Second, the time and spatial integrations are separated. Therefore, when compared with other methods, this new technique can provide substantial improvements (one order of magnitude) in computational speed. However, the benefit of the dimensional reduction is manifested not only in the computation speed but also in the ease of problem set-up since one is required to deal with the boundary not the entire two-dimensional gear geometry. The other benefit of this technique, based on the separation of time and spatial integration, is accuracy. This technique takes the full advantage of the spectral method that has exponential solution convergence. This should be compared to finite element or difference methods where only algebraic convergence is possible.

Since the new technique is extremely efficient, any changes in the time dependent coefficients or source terms in the boundary conditions do not impose a great computational burden on the user. This result is very important when performing accurate Scoring Analysis in gears. This allows the bulk (or blank) temperature to be accurately known. Furthermore, the gear out-of-mesh temperature is not a constant along the tooth profile at steady state running conditions, as is often assumed by gear engineers. The method is also more adaptable for use in small, lubricated, concentrated contacts, such as gears, since high resolution can be obtained without using large numbers of elements.

Currently, we are planning to extend this technique to a multiblock application that includes the remainder of the gear sector and that further optimizes the computation accuracy and speed.

REFERENCES

Burton, A. J. and Miller, G. F., "The Application of Integral Equation Methods to the Numerical Solution of Some Exterior Boundary Value Problems," Proc. R. Soc. A323, 201, 1971.

De Winter, A., and Blok, H., "Fling-Off Cooling of Gear Teeth"; ASME paper 72-PTG-2, 1972.

El-Bayoumy, L. E., Akin, L. S., Townsend, D. P., and Choy, F. C., "The Role of Lubricant Boundary Layers in Transient Thermal Analysis of Spur Gears"; ASME International Power Transmission & Gearing Conference. Vol. 1, pp 313, book no. 10288A, 1989.

H. W. Wyld, "Mathematical Methods for Physics", The Benjamin/Cummings Publishing Co., Inc., Reading, MA.

Koshigoe, S. and Tubis, A., "Wave Structure in Jets of Arbitrary Shape. I. Linear Inviscid Spatial Analysis," Phys. Fluids 29 (12) 3982, 1986.

APPENDIX A

THERMAL ANALYSIS GEOMETRY FOR SPUR GEARS

by

Lee S. Akin * and Dennis P. Townsend **

ABSTRACT

The gear geometry needed to perform a complete thermal analysis for a gear set is very complex and has, to the authors knowledge, never been published before as one set of equations needed for the analysis. The thermal analysis of gears is a very important subject in that it can be used to determine the scoring failure criteria which includes the blank (bulk) temperature and the flash temperature as used in the Blok scoring formula.

1. $$T_t = T_f + T_b$$

The geometry for this analysis must include the involute geometry, the load variance when the teeth mesh is in a single or double tooth contact zone, tooth load sharing due to varying deflection, long and short addendums, tip and/or root relief. Also included are the affect of heat partitioning due to varying tooth contact sliding velocities over constantly changing Hertzian contact band widths, gear set speed and its affect on the lubrication regime (film thickness versus surface roughness).

This geometry analysis will be used as the input parameters to complete the analytical computation of running gear temperatures using Green's function.

INTRODUCTION

This is one more in a series of papers by the authors on their continued study of the art of the prediction of the onset of scoring or scuffing in high performance gear drive mechanisms and our attempt to make it more scientific. These studies have examined a series of disciplines from an interdisciplinary lubrication theory (ref. 1), to a study of the effect of windage on the lubricant flow into high speed gear teeth (Ref. 2) and a model for lubricant flow in between gear teeth (Refs. 3, 4, 6, 7, 10 and 11, 13, 16).

In addition these studies have evaluated the analytical and experimental spur gear temperature effects on operating variables (Ref. 5), gear lubrication and cooling studies (Refs. 7 to 9, and 12), investigations of affect of transient (time variant) thermal and lubricant boundary layers (Refs. 14 and 15).

Most of the above and all of the thermal work was done using finite element methods which produce large matrixes causing slow computer solutions to provide satisfactory accuracy. This geometry analysis is developed for, "a computer program for the computation of running gear temperatures using Greens function" provides a new and unique solution to the gear temperature analysis problem. The above is accomplished by using special integration techniques newly developed to accommodate the special conditions found in lubricated concentrated sliding contacts, such as found in high performance aircraft gear drives. This appendix describes the intricate gear geometry needed for this analysis, mostly available in the literature but certainly not all in one place. This paper hopes to fill that need.

FORMULATION OF GEOMETRICAL EQUATIONS

The equations for the involute curve in rectangular coordinates is shown in equation set 2 (see Fig. 1).

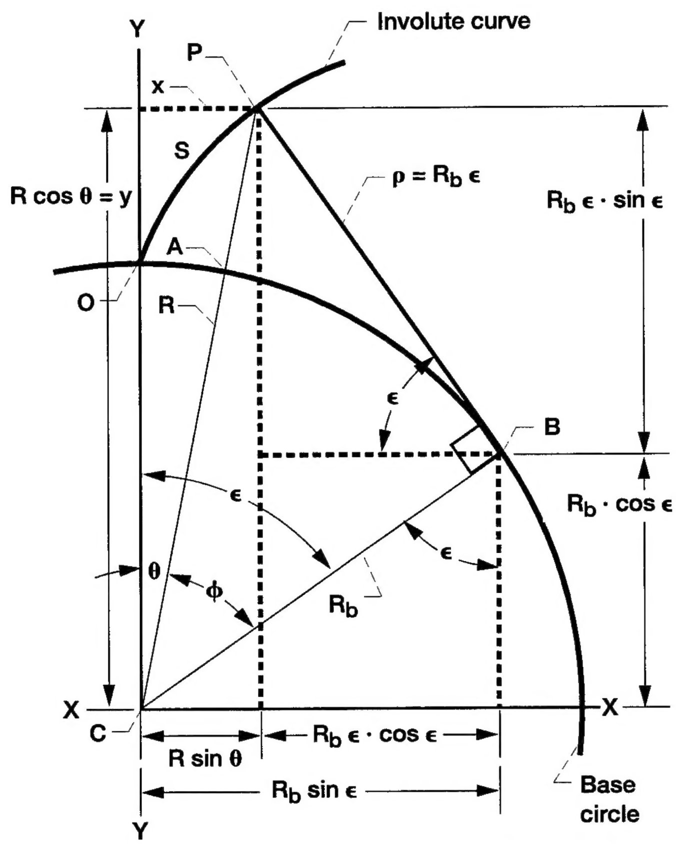

Figure 1.—Involute curve geometry.

2. $x = (\cos\theta + \theta\sin\theta)\,R$ or $x = R\sin\theta$ see nomencalture

$$y = (\sin\theta + \theta\cos\theta)R_b \quad \text{or} \quad y = R\cos\theta$$

$$x^2 + y^2 = (1+\theta^2)^{1/2}\frac{1}{R_b}$$

* Gearesearch Associates. ** NASA-Lewis Research Center.

The polar/vectorial angle θ, in Fig. 1 may be calculated from:

$$\theta = \left(x^2 + y^2 - R_b^2\right)^{1/2} \frac{1}{R_b} = \frac{\left(R^2 - R_b^2\right)^{1/2}}{R_b}$$

3. $$-\tan^{-1}\left[\frac{\left(R^2 - R_b^2\right)^{1/2}}{R_b}\right] = \tan\phi - \phi = \mathrm{inv}\phi$$

where: $R = \frac{N}{2P}$ & $R_o = \frac{N+2}{2P}$ & $R_b = R\cos\phi$

so that the pressure angle ϕ in Fig. 1 is:

4. $\theta = \tan^{-1}\frac{\left(R^2 + R_b^2\right)^{1/2}}{R_b} = \cos^{-1}\frac{R_b}{R}$ and the roll angle is $\varepsilon = \theta + \phi = \tan\phi$. The important radii of curvature equations, some at critical locations, are shown below in equation set 5, and see Figs. 1 and 2:

5. $\rho = R\sin\phi = \varepsilon R_b$ and for the mating gear $\rho_m = R_m \sin\phi = \varepsilon_m R_{bm}$

The lowest point of contact for the mesh is ρ_c calculated from:

$$\rho_c = C\sin\phi - \left(R_o^2 + R_b^2\right)^{1/2}$$

and

$$\rho_{cm} = \left(R_{om}^2 + R_{bm}^2\right)^{1/2}$$

$$R_c = \left[\left(C\sin\phi - \sqrt{R_o^2 + R_b^2}\right)^2 + R_b^2\right]^{1/2}$$

where $C\sin\phi = L_a$ the line of action, and

$$R_{cm} = \left[\left(C\sin\phi - \sqrt{R_{om}^2 + R_{bm}^2}\right)^2 + R_{bm}^2\right]^{1/2}$$

is the lowest point on the mating gear. The radii of curvature at the lowest and highest points of single tooth contact may be calculated from:

$$\rho_l = \left(R_o^2 + R_b^2\right)^{1/2} \qquad R_l = \left(\rho_l^2 + R_b^2\right)^{1/2}$$

$$\rho_h = C\sin\phi - \left(R_{om}^2 - R_{bm}^2\right)^{1/2} + p_b$$

$$R_h = \left(\rho_h^2 + R_b^2\right)^{1/2}$$

where the base circle pitch is $p_b = \frac{2\pi}{N}\cos\phi$ and the circular pitch

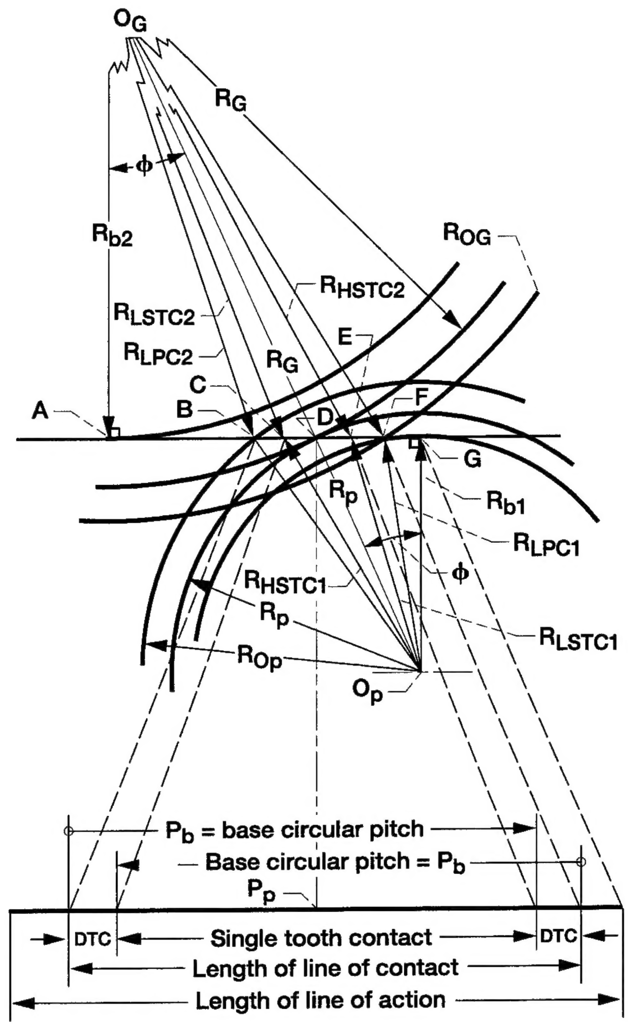

Figure 2.—Gear mesh line of action geometry.

is $p = \frac{\pi D}{N}$. The distance of roll/slide S along the involute curve may be calculated by integrating over the roll angle ε, from (vogle, Ref. 17), see Fig. 1:

6. $$S = \int_{\varepsilon_1}^{\varepsilon_2} R_b \varepsilon d\varepsilon = \frac{R_b}{2}\varepsilon^2\Big|_{\varepsilon_1}^{\varepsilon_2} = \frac{R_b}{2}\left(\varepsilon_2^2 - \varepsilon_1^2\right)$$

and since, as can be noted from Fig. 1, $\rho = R_b\varepsilon = R_b\tan\phi$ and $\varepsilon = \frac{\rho}{R_b}$ thus $S_{1-2} = \frac{\rho_2^2 - \rho_1^2}{2R_b}$ over any arbitrary portion of the profile with subscripts 1 and 2 and $S_{c-o} = \frac{\rho_0^2 - \rho_c^2}{2R_b}$ over the whole profile from the first point of mesh contact on the profile at S_c to the last point at outside diameter S_o. For example: from $\overline{R}_c$ to R_l to R (at pitch point) to R_h to R_o at the outside radius. The arc length along the base circle Σ can be

calculated from (were subscripts 1 and 2 are at arbitrary locations on the tooth profile) as shown.

7. $\Sigma_{1-2} = R_b(\varepsilon_2 - \varepsilon_1) = R_b\ (\tan\phi_2 - \tan\phi_1)$so that $\Sigma_{1-2} = R_b\left(\frac{\rho_2}{R_b} - \frac{\rho_1}{R_b}\right) = \rho_2 - \rho_1$ and thus the ratio between the distance S and the distance Σ becomes $k = \frac{\rho_2 + \rho_1}{2R_b}$ a quantity useful in thermal calculations. The transition time, as a function of radii of curvature and its rolling velocity between any two points along the tooth, can be expressed in equation set 8 as:

8. $$V = \frac{d\rho}{dt} = \frac{d\varepsilon}{dt}R_b = R_b\omega\frac{rad}{sec}$$

where:

$\frac{d\varepsilon}{dt} = \omega$, a constant and since $\rho = \varepsilon R_b$ then $d\varepsilon = \omega dt$ and $\frac{\omega}{\varepsilon}dt = \frac{d\rho}{\rho}$, so that $\int_{\rho_1}^{\rho_2}\frac{d\rho}{\rho} = \ln\frac{\rho_2}{\rho_1} = \frac{\omega}{\varepsilon_2 - \varepsilon_1}(t_2 - t_1) = \frac{\omega(t_2 - t_1)}{\frac{1}{R_b}(\rho_2 - \rho_1)}$ and $\Delta t_{1,2} = (t_2 - t_1) = \frac{\rho_2 - \rho_1}{R_b\omega}\ln\frac{\rho_2}{\rho_1}$ is the time it takes for the gear to rotate from ε_1 to ε_2. The critical dimensions along the line of action are shown in figure 2 and described below in equation 9:

9. $Z = \sqrt{R_{o1}^2 - R_{b1}^2} + \sqrt{R_{o2}^2 - R_{b2}^2} - C\sin\phi$ is the length of the "line of contact b – f" as a subset of the "line of action a – g."

Now we can calculate the width of the Hertzian band of mutual contact at the mesh point from equation 10: (Timoshenko Ref.18)

10. $B = \left(\frac{16W_n(K_1 + K_2)\rho_1\rho_2}{F(\rho_1 + \rho_2)}\right)^{1/2}$ where: $K_1 = \frac{1 - v_1^2}{\pi E_1}$ and $K_2 = \frac{1 - v_2^2}{\pi E_2}$ and $K_1 + K_2 = \frac{2}{\pi}\left(\frac{1 - v^2}{E}\right)$ and since: $\rho_1 + \rho_2 = L_a = C\sin\phi$ and $W_n = W_t/\cos\phi = W_t\sec\phi$, see Fig. 3, and $E_1 = E_2 = E$

$$B = 3.19\sqrt{\frac{W_t(1 - v^2)}{F\cos\phi C\sin\phi E}}(\rho_1\rho_2)^{1/2}$$

The rolling velocity for the gear V_1 and it's mating gear V_2 are calculated from equation set 11:

11. $V_1 = \frac{n_1\pi\rho_1}{360}$ ft/sec and $V_2 = \frac{n_2\pi\rho_2}{360}$ ft/sec, so that the sliding velocity $V_s = V_1 - V_2 = \frac{\pi}{360}(n_1\rho_1 - n_2\rho_2)$ ft/sec where: $V_1 = V$ & $V_2 =$ Vm.

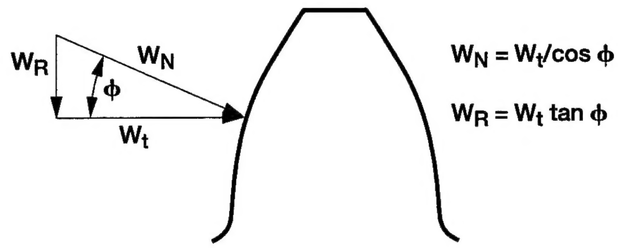

Figure 3.—Gear tooth profile showing normal load W_N.

Therefore the rolling velocity anywhere along the profile can be calculated from: $V_1 = \frac{\pi\rho_i}{114.59}$ ft/sec and $V_m = \frac{n_m\rho m}{114.59}$ ft/sec for the mating gear so that the sliding velocity is calculated from: $V_{is} = \frac{n\rho_i + n_m\rho_{im}}{114.59}$ ft/sec where n = speed of gear and n_m = the speed of the mating gear. Another value needed to calculate the coefficient of friction is the total velocity from: $V_t = \frac{n\rho_i + n_m\rho_{im}}{114.59}$ ft/sec.

We can now calculate the coefficient of friction as:

12. $f = 0.0127\log_{10}\left(\frac{3.17\times10^6 W_t\sec\phi}{F\mu_{cp}V_sV_t}\right)$ where F = tooth face width, μ_{cp} = viscosity in centipoise and W_t = tangential tooth load in lbs. Thus we find ourselves in a position to calculate the instantaneous heat flux $q(\rho)_I$ as a function of the radius of curvature at the instantaneous position "i" along the line of contact per equation Set 13.

13. $$q(\rho_i) = \frac{W_t\sec(\phi)f(n\rho_i - n_m\rho_{im})}{170\,330}\ \text{BTU/min}$$

This equation can also be written in a form more useful using the pinion speed only:

$q(\rho_i) = \frac{W_t\sec(\phi)n_pf}{170\,330}(\rho_{pi} - m_g\rho_{gi})$ BTU/min, where $m_g = \frac{N_g}{N_p}$ the gear ratio. At times it is more convenient to calculate the hear flux from the radius vector at the instantaneous contact points where:

$$qR_i = \frac{W_t\sec(\phi)n_pf}{170\,330}\left\{C\sec\phi(1 - m_g) - \left(R_{gi}^2\Big|_{R_{gLPC}}^{R_{go}} - R_{gb}^2\right)^{1/2} + m_g\left(R_{pi}^2\Big|_{R_{pLPC}}^{R_{po}} - R_{pb}^2\right)^{1/2}\right\}\ \text{BTU/min}$$

CLOSURE

This appendix develops the geometry analysis for the input for the computer solution of the thermal analysis of spur gears using Green's function to solve for gear blank surface temperatures. This

geometry analysis provides the gear tooth geometry input, the gear tooth sliding distance parameters, the rolling and sliding velocity inputs and the equations for the frictional heating developed during the gear tooth meshing, as a function of the location on the gear tooth.

Using these inputs the program can then determine the transient and steady state temperatures of the gear teeth.

REFERENCES—by L.S. Akin and Dennis P. Townsend et.al.

1. An Interdisciplinary Lubrication Theory for Gears - 1973, ASME - Jrnl. of Engr/g for Ind'y, Vol. 95, Series B, no. 4.

2. Theory for the Effect of Windage on the Lubricant Flow in Tooth Spaces of Spur Gears - 1975, Jrnl of Engr'g, Vol. 97, Series B, no. 4.

3. Study of Lubricant Flow Phenomena in Spur Gears - 1975, ASME - Jrnl of Lub. Tech'y, Vol. 97, Series F, no. 2.

4. Study of Lubricant Flow Phenomena in Spur Gears - Out-of-Mesh Condition, AMSE, - Jrnl of Mech Design, no. 1.

5. Analytical and Experimental Spur Gear Temperature as affected by Operating Variables - 8/1980, ASME - 3rd inter. PT & Gear Conf., Jrnl of Mech Design, Vol. 100, no. 1.

6. Oil Jet Flow Phenomena in Spur Gears - Results from Latest Studies - 1980, ASME - Western Design Tech. Conf., 80-C2/DET-34.

7. Cooling of Spur Gears with Oil Jet Directed Into the Engaging Side of Mesh at Pitch Point - 1981, JSME - Proc'gs of Inter. Symp. on gearing & pwr. trans., Paper B-4, pg. 253-260. Also 6/81, NASA-Lewis Sym. on Adv'cd Power trans. Tech.

8. Gear Lubrication and Cooling Experiments and Analysis - JSME - Proc. of Inter. Sym. on Gearing and Pwr Trans. - 1981, Paper B-3, pg. 253-260.

9. Experimental and Analytical Determination of gear Tooth Temperature with Oil Jet Lubrication - 1981, AGARD Conference. Proc. No. 323, Neuilly Sur Sein, France.

10. Into Mesh Lubrication of Spur Gears with Arbitrary Offset Oil Jet - Part 1 - For Jet Velocity Less Than or Equal to Gear Velocity - ASME, WAM, 11/82 - Jrnl Mech. Trans. and Auto'tn in Design, Vol. 105, pg 713, Dec. 83.

11. Into Mesh Lubrication of Spur Gears with Arbitrary Offset Oil Jet - Part 2 - For Jet Velocities Equal to or Greater than Gear Velocity - ASME, WAM,11/82. Also Jrnl Mech. Trans.. and Auto'tn Design, Vol. 105, pg 719, Dec. 83.

12. Parametric Studies of Gear Cooling Using Automatic FEM Generator - ASME, DET Conf., Oct. 84, 84-det-155 and Jrnl Mech. Trans. and Auto'tn Design, Vol. 107, 1985.

13. Lubricant Jet Flow Phenomena in Spur Gears with Modified Center Distances and/or Addendums - For Out of-Mesh Conditions - ASME, 84-DET-96 and Jrnl Mech. Trans. and Auto'tn Design, Vol. 107, pg 24, 1985.

14. An Investigation of the Transient Thermal Analysis of Spur Gears - ASME, 84-DET-92 and Jrnl Mech and Auto'tn Design, Vol. 107, no. 2, 1985.

15. The Roll of Thermal and Lubricant Boundary Layers in Transient Thermal Analysis of Spur Gears - ASME, Inter'l Power Transmission and Gearing Conference, Vol. I, pg 313, book no. 10288A, 1989.

16. Lubricant Jet Flow Phenomena in Spur and Helical Gears with Modified Addendums - For Radially Directed Individual Jets - ASME, Inter. Power Transmission and Gearing Conference, Vol. I, pg 367, book 10288A,1989

17. Involutometry * Trigonometry - seven place tables of natural funcitons by Dr. Werner F. Vogel, 1945 Michigan Tool Co.

18. Theory of Elasticity, Pg 382, by Timoshenko & Goodier, 1951.

NOMENCLATURE FOR APPENDIX A

B	width of the band of mutual contact at the mesh point
C	center distance for mating pair
E	Young's modulus of elasticity
F	face width
f	coefficient of friction
M_g	gear ratio N_g/N_p
N	number of teeth in gear and N_m number of teeth in the mating gear
n	pinion speed, rpm, n_m for mating gear and n_p for pinion
P	diametral pitch of the teeth
p_b	circular pitch of gear and its mating gear
qR_t	heat flux calculated using the instantaneous radius vector
$q(\rho_1)$	heat flux due to sliding friction at instantaneous point of contact using the radius of curvature as a parameter
R	radius vector to the pitch point and R_m for the mate at the contact point
R_b	base radius of the involute curve (its origin) and R_{bm} for mating gear
R_c	radius vector to lowest point of contact from center of gear and R_{cm} for mating gear
R_{gb} & R_{pb}	base radius of gear and pinion respectively
R_{gi} & R_{pi}	instantaneous radius og gear and pinion respectively
R_{go} & R_{po}	outside radius of gear and pinion respectively
R_{LPC2} & R_{LPC1}	lowest point of contact for gear and pinion respectively
R_o	outside radius of the gear and end of the involute curve and R_{om} for mating gear
R_l & R_h	radius vector to lowest and highest points of single tooth contact
V	velocity along the curve (involute)
V_s	the sliding velocity at an instantaneous point $v_1 - v_2$ in the mesh
V_t	the total velocity at an instantaneous point in the mesh $v_1 + v_2$
V_l	rolling velocity of the gear and its mating gear
W_t & W_n	tangential and normal (perpendicular) load, respectively
x & y	Cartesian coordinates of the involute curve from its origin at the base circle
Z	length of the line of contact as a subset on the line of action
$\Delta t_{1,2}$	the time it takes to rotate from ε_1 to ε_2
ΔN	virtual number of teeth expansion or reduction for long and short addendums
ε	roll angle on tooth
θ	involute polar angle = $\text{inv}\phi = \tan\phi - \phi = \varepsilon - \phi$ (rad)
μ_{cp}	oil viscosity, in cp
ρ	radius of curvature from the base circle and ρ_m for the mating gear at contact point
ρ_c	radius of curvature at lowest or initial point of contact from base circle and ρ_{cm} for mating gear at contact point
ρ_l & ρ_h	radius of curvature at lowest and highest points of single tooth contact
ϕ	pressure angle of mesh

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY (*Leave blank*)

2. REPORT DATE
September 1996

3. REPORT TYPE AND DATES COVERED
Technical Memorandum

4. TITLE AND SUBTITLE
A Computer Program for the Computation of Running Gear Temperatures Using Green's Function

5. FUNDING NUMBERS
WU–505–62–36
1L162211A47A

6. AUTHOR(S)
S. Koshigoe, J.W. Murdock, L.S. Akin, and D.P. Townsend

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
NASA Lewis Research Center
Cleveland, Ohio 44135–3191
and
Vehicle Propulsion Directorate
U.S. Army Research Laboratory
Cleveland, Ohio 44135–3191

8. PERFORMING ORGANIZATION REPORT NUMBER
E–10286

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
National Aeronautics and Space Administration
Washington, D.C. 20546–0001
and
U.S. Army Research Laboratory
Adelphi, Maryland 20783–1145

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
NASA TM–107241
ARL–TR–1117

11. SUPPLEMENTARY NOTES
Prepared for the Seventh International Power Transmission and Gearing Conference sponsored by the American Society of Mechanical Engineers, San Diego, California, October 6–9, 1996. S. Koshigoe, J.W. Murdock, and L.S. Akin, Gearesearch Associates, 750 Indian Wells Road, Banning, California 92220–5308; D.P. Townsend, NASA Lewis Research Center. Responsible person, D.P. Townsend, organization code 2730, (216) 433–3955.

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Unclassified - Unlimited
Subject Category 37

This publication is available from the NASA Center for AeroSpace Information, (301) 621–0390.

12b. DISTRIBUTION CODE

13. ABSTRACT ***(Maximum 200 words)***
A new technique has been developed to study two dimensional heat transfer problems in gears. When compared with the finite element or finite difference methods, this new technique can provide substantial improvements in computational speed. It is also more adaptable for use in small lubricated concentrated contacts, such as gears, since high resolution can be obtained without using large numbers of nodes or elements. The new technique consists of transforming the heat equation into a line integral equation with the use of Green's theorem. The equation is then expressed in terms of eigenfunctions that satisfy the Helmholtz equation, and their corresponding eigenvalues for an arbitrarily shaped region of interest. It is shown that the discretized version of the eigenfunction can be obtained by solving a set of homogenous simultaneous equations. Once the eigenfunctions are found, the transient temperature is expanded in terms of the eigenfunctions with unknown time dependent coefficients that can be solved by using Runge-Kutta methods. The time integration is extremely efficient, therefore, any changes in the time dependent coefficients or source terms in the boundary conditions do not impose a great computational burden on the user. This result is very important when performing accurate scoring analysis in gears so that the gear temperature can be accurately determined. The gear, out of mesh surface temperature is not a constant along the tooth profile at steady state running conditions as is usually assumed by gear engineers.

14. SUBJECT TERMS
Gears; Green's function; Heat transfer; Cooling

15. NUMBER OF PAGES
11

16. PRICE CODE
A03

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	